



2024 Academic Challenge

MATH TEST – SECTIONAL

Math Production Team

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GENERAL DIRECTIONS

Please read the following instructions carefully. This is a timed test; any instructions from the test supervisor should be followed promptly.

The test supervisor will give instructions for filling in any necessary information on the answer sheet. Most Academic Challenge sites will ask you to indicate your answer to each question by marking an oval that corresponds to the correct answer for that question. One oval should be marked to answer each question. Multiple ovals will automatically be graded as an incorrect answer.

Be sure ovals are marked as  , not  ,  ,  , etc.

If you wish to change an answer, erase your first mark completely before marking your new choice.

You are advised to use your time effectively and to work as rapidly as you can without losing accuracy. Do not waste your time on questions that seem too difficult for you. Go on to the other questions, and then come back to the difficult ones later if time remains.

Time: 40 Minutes Number of Questions: 30

DO NOT OPEN TEST BOOKLET UNTIL YOU ARE TOLD TO DO SO!

**2024 Academic Challenge
Sectional Mathematics Exam**

1. A new product is revealed gradually to a company employing 1,200 people. Every three days, one-fourth of the workforce learns about the product. After 10 days, how many people are still unaware of the product?
a) 250 b) 360 c) 460 d) 500 e) 540

2. If $\log_3 25 = x$ and $\log_3 16 = y$, what is $\log_3 20$?
a) $(x - y)/2$ b) $(x + y)/2$ c) $x - y$ d) $x + 2y$ e) $x + y$

3. If 35 percent of students carry index cards, 47 percent carry sticky notes, and 19 percent carry index cards and sticky notes, what percent of students carry index cards or sticky notes?
a) 12 b) 54 c) 63 d) 66 e) 82

4. Each side measure of a regular octagon is 12 centimeters. What is the area of the octagon?
a) 695.28 cm^2 b) 1420.96 cm^2 c) 521.46 cm^2 d) 1390.56 cm^2 e) 764.36 cm^2

5. What is the slope of the tangent line to the equation $2x^2 + 3xy = 18$ at the point (3,0)?
a) $-3/4$ b) $-4/3$ c) $2/3$ d) $3/4$ e) $9/2$

6. If a sequence starts with the terms 14, 19, 24, 29 ..., what will the 15th term of the sequence equal?
a) 59 b) 64 c) 74 d) 84 e) 89

7. If an object travels 4,800 degrees around the unit circle, how many revolutions has it completed?
a) 10 b) 12 c) 13 d) 14 e) 15

8. If the volume of a pyramid with a square base must hold at least 800 cubic meters, what do the dimensions need to be?

- a) base with each side 12 meters; height of 15 meters
- b) base with each side 10 meters; height of 22 meters
- c) base with each side 15 meters; height of 10 meters
- d) base with each side 13 meters; height of 15 meters
- e) base with each side 12 meters; height of 16 meters

9. Solve for x . $3^{2x+4} = 81^{6x+12}$.

- a) -3
- b) -2
- c) 2
- d) 3
- e) 4

10. During February, the probability of snow during a bus route is 32 percent. A bus is late to its stop 11 percent of the time if there is no snow. When there is snow, the bus is late 27 percent of the time. Given that the bus arrives on time, what is the probability there is snow?

- a) 0.2155
- b) 0.2500
- c) 0.2635
- d) 0.2785
- e) 0.3215

11. What are the roots of $x^2 - 4x + 27$?

- a) $4 \pm \sqrt{23}$
- b) $1 \pm i\sqrt{23}$
- c) $4 \pm i\sqrt{23}$
- d) $2 \pm \sqrt{23}$
- e) $2 \pm i\sqrt{23}$

12. For the function $|(x + 2)|$, what is the value of the derivative at $x = -2$?

- a) 0
- b) 1
- c) -1
- d) The function is not continuous at $x = -2$.
- e) The function is not differentiable at $x = -2$.

13. Consider the following piecewise function:

$$f(x) = \begin{cases} x^2 - 4x, & x < 6 \\ \sqrt{bx + 3}, & x \geq 6 \end{cases}$$

What value of b would make the function continuous?

- a) 3 b) 4.5 c) 12.5 d) 23.5 e) 25

14. A parabola has a focus at (2, 6). The directrix for the parabola is at $x = -8$. What is the equation of the parabola?

- a) $y = 1/20(x + 3)^2 + 6$
b) $x = 1/20(y - 6)^2 - 3$
c) $x = 1/20(y - 2)^2 + 3$
d) $y = 20(x + 3)^2 - 6$
e) $x = 20(y - 6)^2 + 3$

15. A card player is holding a five-card set. Three of the cards are clubs, with the highest rank of 10. (The ace is the high card in this game.) Assume that three other players each hold 5 other cards, but none of them is a club with a rank above 10. If the player draws a card from the deck, what is the probability of getting a club above 10? Assume the full, 52-card deck is used for this game.

- a) 2/5 b) 4/13 c) 1/13 d) 1/8 e) 3/13

16. A student draws a rectangle with a width of 7 inches and a length of 6 inches on a standard letter-size page of 8.5 inches by 11 inches. If another student, without looking, randomly puts a dot on the paper, what is the probability the dot will be within the rectangle?

- a) 0.4017 b) 0.4492 c) 0.5218 d) 0.5414 e) 0.5819

17. A vehicle travels east at an average speed of 50 miles per hour. Two hours later, a second vehicle, starting from the same point as the first vehicle, travels north at an average speed of 45 miles per hour. After the second vehicle has been traveling for 3 hours, how far apart will the two vehicles be?

- a) 156.6 miles b) 212.7 miles c) 235.4 miles d) 284.1 miles e) 312.3 miles

18. Two angles form a linear pair. One angle has a measure of $x^2 + 28x$, while the other angle has a measure of $29x$. What is the value of x ?

- a) 60 b) 45 c) 3 d) -4 e) -60

19. A sound barrier is supported by a series of pillars. The first pillar is 5 feet tall, and the height of the pillars increases by 3 feet at each stage. If there are 40 pillars, what is the sum of the height of the pillars?

- a) 600 feet b) 625 feet c) 2537 feet d) 2540 feet e) 2543 feet

20. A regular polygon has an exterior angle measuring 18 degrees. What is the sum of the interior angle measures of the polygon?

- a) 1440 b) 2160 c) 2880 d) 3040 e) 3240

21. A series starts with the following numbers:

18, $27/2$, $81/8$, $243/32$...

What is the sum of the series?

- a) $27/4$ b) 36 c) $9/2$ d) 72 e) $27/2$

22. Two classrooms each have 34 students. The mean height of Classroom A is 64 inches, and the standard deviation is 1.3 inches. The mean height of Classroom B is 67 inches, and the standard deviation is 2 inches. If the two classes are combined, what are the mean and the standard deviation of the height of the students?

- a) mean of 65.5 inches and standard deviation of 3.3 inches
b) mean of 65.5 inches and standard deviation of 2.4 inches
c) mean of 65.5 inches and standard deviation of 1.7 inches
d) mean of 65 inches and standard deviation of 2.2 inches
e) mean of 65.5 inches and standard deviation of 5.7 inches

23. A fruit stand expects to sell twice as many oranges as apples. If the stand can process no more than 160 orders during the lunch hour, and it earns a profit of 15 cents per apple and 20 cents per orange, what is the maximum profit it can earn during the lunch hour?

- a) \$19.00 b) \$21.45 c) \$25.75 d) \$29.05 e) \$29.15

24. If a polyhedron has 12 faces and 16 vertices, how many edges will it have?

- a) 18 b) 22 c) 24 d) 26 e) 28

25. Which function yields a maximum output of $y = 2$ and a minimum output of $y = -5$?

- a) $y = 2 \sin x - 5$
- b) $y = 2 \cos x - 5$
- c) $y = 5 \cos x - 2$
- d) $y = 5 \sin x - 2$
- e) $y = 3.5 \sin x - 1.5$

26. A right triangle has integer side measures. Its perimeter and its area are the same value. What is the smallest possible sum of the legs?

- a) 2
- b) 4
- c) 7
- d) 14
- e) 22

27. A square floor tile is expected to have a side measure of 5 inches. If each side has a measuring error of $1/50$ inch, what is the expected error in the area of the tile?

- a) $1/50$ inch
- b) $1/25$ inch
- c) $1/10$ inch
- d) $1/5$ inch
- e) $1/2$ inch

28. If the polynomial $2x^3 - 6x^2 - 12x + 16$ is factored, what are its roots?

- a) -1, 3, -5.
- b) -2, 1, 4.
- c) 2, $1/2$, $3/2$.
- d) -3, -1, $2/3$.
- e) $1/4$, $3/4$, $-3/4$

29. A student calculates the value of a log expression (written as $y = \log_b x$) as a negative number. Assuming the expression cannot return a value of zero, what must be true about the expression?

- a) $b < 0$
- b) $x < 0$
- c) $x = 0$
- d) $0 < x < 1$
- e) $x = 1$

30. For the graph of the polar equation $r = 4 \sin 3\theta$, what is the best description of the maximum and minimum values of the distance from the x -axis?

- a) 2 positive, equal maximum values; 2 negative, equal minimum values
- b) 2 positive, equal maximum values; 1 negative minimum value
- c) 2 positive, distinct maximum values; 2 negative, distinct minimum values
- d) 2 positive, distinct maximum values; 1 negative minimum value
- e) 2 maximum values of zero; 1 negative minimum value

SCRATCH PAPER

